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VIII-A: Finite affine matching certificates for a spinless conical spacetime

Landmark VIII-A

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Abstract

We formulate a finite admissibility interface for polygonal slices of a declared 2 + 1-dimensional spinless conical spacetime. Geometric realization is checked before affine matching. The matching packet separates vertex and overlap equations, particle data, topologically marked holonomy words, collar agreement, and pointwise worldline fixation.

In a fixed developed presentation, incompatible proposed values of a prescribed transition cannot both be certified. A deterministic controller that factors through a named surviving observable algebra gives equal certified geometric outputs on equal accessible inputs, provided its complete control and resource specification is spectator blind. We prove the corresponding spectator-extension and restricted deferred-readout statements locally. An instrument-level obstruction shows why equal classical record laws do not determine residual source functionals.

For the fixed cone of deficit $\pi/2$, we give an explicit uniformly spacelike, embedded, acausal graph family with exactly one incidence event: the particle link changes from three faces to six. Exact vertices, lengths, areas, adapted affine charts, primitive winding, and full affine peripheral products furnish the certificate on both one-sided germs. The persistent particle signature is preserved, although the realization-dependent matching tables change. The intrinsic particle angle defects are computed separately and are not identified with the spacetime

deficit.

Two incomplete updates fail different obligations. A changed deficit with a stale spatial rotation fails seam matching while still fixing the stationary worldline. A time-shifted rotation fails pointwise worldline fixation. The results establish an admissibility layer, not a sourced backreaction law, canonical polygon evolution, quantum recovery mechanism, or spacetime emergence theorem.

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Introduction and scope

An observation boundary specifies which state data a controller may use. It does not specify which geometric outputs are admissible. A geometric coupling therefore requires two independent pieces: a declared accessible input and a complete admissibility certificate for its output.

This paper supplies a finite matching contract for one declared spinless conical model. The notation Ein_0, retained

from the VIII programme, denotes that contract. It is not a classification of arbitrary Einstein initial data and is not an unstated continuum Einstein equation.

The classical geometric motivation is that source-free 2 + 1-dimensional Einstein geometry with zero cosmological constant is locally flat [1]. Polygonal and holonomy descriptions have a first-order-gravity interpretation [2]. These references motivate the dictionary. The dictionary itself is stated explicitly below, and no unquoted constraint-algebra theorem is used.

Our polygonal descriptions need not belong to a particular trivalent canonical polygon gauge. Nonconvex faces and regular vertices of valence four are allowed. We prove matching statements for the interface actually defined here.

Principal results

The four principal statements are:

1. *Fixed-presentation completeness.* Once the developed realization, adapted charts, overlap components, reference framing, and seam maps are fixed, two different proposed values of a required transition cannot both pass the contract.

2. *Constraint-blindness.* Equal complete accessible inputs give equal applicability, geometric outputs, evaluated packets, and deterministically selected certificates.

3. *Holonomy reduction.* After realization and prescribed-transition checks, the holonomy obligations on a marked punctured disk reduce to regular identity relators, one primitive positive peripheral word, and its collar agreement.

4. *Independent rejection tests.* A stale deficit rotation fails deficit/seam matching. A time-shifted rotation fails the spinless pointwise-axis condition. Neither failure can replace the other.

The worked fixed-cone certificate is the substantive geometric instance of the contract. A1 and A2 are deliberately short: they are uniqueness in a prescribed presentation and factorization through declared inputs, respectively. Neither is advertised as a classification or an existence theorem for sourced dynamics.

Relationship to companion manuscripts

The observation-boundary and residual-instrument questions are compared with Cheung's VII-E [14]. The fixed three-to-six slicing is compared with VII-F [17]. Every identity needed in this paper is proved here.

The recovery distinctions are compared with Oai280-C and Hawk04-B [21, 27]; they do not add a decoder to the geometric interface. VIII-B and VIII-C are separate sourced-realization and composition projects [18, 19]. Their theorems are not imported.

Companion identifiers distinguish models and documentary relationships. Similar terminology does not identify their equation spaces, observable algebras, channels, cutoffs, or geometric dictionaries.

toprule Object Status in this paper

midrule Finite spinless matching contract Defined, with realization prerequisites

A1–A3 Proved under their explicit hypotheses

Fixed-$\pi/2$ family and unique incidence event Constructed and certified locally

Instrument residual obstruction Proved locally

Spectator and restricted deferred comparisons Proved for the declared interfaces

Stale-deficit and time-shifted proposals Rejected by explicit calculations

Variable-deficit sourced constructor Not constructed here; separate VIII-B object

Canonical polygon flow or a hybrid history Not constructed here; separate VIII-C objects

Mixed incidence–deficit germ Not certified

Quantum characteristic coefficient or field source Not constructed or admitted

Holographic reconstruction or spacetime emergence Not claimed

bottomrule

Matter interfaces and the observation boundary

Finite states and destructive readout

For a finite labelled register Γ, let $\mathscr{H}_\Gamma$ be its finite-dimensional Hilbert space. Write $L(\mathscr{H}_\Gamma)$ for its operators and $\mathscr{D}(\mathscr{H}_\Gamma)$ for its density operators. A channel is a completely positive, trace-preserving linear map.

Quantum names are access rights to factors of one joint state. Classical registers have distinguished bases. Admissible states are jointly block diagonal in those bases, including their correlations with all other registers. A diagonal classical marginal alone is not the required condition.

Born destructive computational-basis readout is a specified primitive, in the sense of the indexed Enter calculus and of the Philosophy of Enter essay [8, 9]. For a measured factor S, complement B, and classical output record R, put

$$M_z = \langle z|_S \otimes I_B.$$

Its channel is

$$\mathscr{M}_S(X) = \sum_z \mathrm{ketbra}\, zz_R \otimes M_z X M_z^\dagger, \qquad \mathscr{M}_S : L(\mathscr{H}_{SB}) \to L(\mathscr{H}_{RB}).$$

The factor S is absent from the output. No uniqueness theorem for the Born law is needed as a premise.

Coherent recording instead uses the isometry

$$V(|z\rangle_S \otimes |b\rangle_B) = |z\rangle_S \otimes |z\rangle_A \otimes |b\rangle_B,$$

where A is a fresh quantum carrier. It retains both S and A. A nondestructive instrument is another interface and must be specified separately.

Lemma (Readout, forgetting, and consumed names). *On every operator, including operators with an untouched reference,*

$$\mathrm{Tr}_R \mathscr{M}_S(X) = \mathrm{Tr}_S X.$$

Every output source observable acts on declared surviving output names. An operator on the consumed factor S is not a surviving source operator unless a new output representation is explicitly supplied.

Proof. Expand

$$X = \sum_{z,z'} \mathrm{ketbra}\, zz'_S \otimes X_{zz'}.$$

Readout returns $\sum_z \mathrm{ketbra}\, zz_R \otimes X_{zz}$. Tracing R gives $\sum_z X_{zz} = \mathrm{Tr}_S X$. An untouched reference can be included in B.

The codomain in contains no S-factor. A pullback through $\mathscr{M}_S^*$ is an input observable used to calculate an output expectation, not a restored output factor. A representation of an S-observable on RB would be additional interface data.

Operational forgetting is not physical annihilation of a complete dilation. It also does not determine a heat, work, or memory-reset cost; compare [23].

Named surviving source algebra

Definition (Named source boundary). A source boundary consists of a unital ^*-subalgebra

$$\mathscr{A}_{\mathrm{src}} \subseteq L(\mathscr{H}_{\Gamma_{\mathrm{src}}})$$

on declared surviving names, together with a specified embedding into the complete current output interface. Its accessible state input is

$$\omega_\rho|_{\mathscr{A}_{\mathrm{src}}} : B\mathrm{longmapsto}\,\mathrm{Tr}\,(\rho B).$$

Classical locations enter only through a declared classical-register encoding or as explicitly declared geometric/controller data.

This boundary does not automatically include consumed factors, inaccessible carriers, spectator cardinality, implementation history, or coherences outside the named algebra. A produced record may be accessible when explicitly authorized. That choice changes the applicable comparison theorems.

Complete instrument agreement versus record agreement

A finite instrument is a family of completely positive maps $\{\mathcal{I}_w\}_w$ whose sum is trace preserving. Its effects are $E_w = \mathcal{I}_w^*(I)$. Its recorded channel returns the classical outcome and the residual output.

Let $\{P_z\}_z$ be a projective measurement on an input space H, let q aggregate its fine outcomes, and let $T : L(H) \to L(K)$ be a channel. Define

$$\Delta_w(X) = \sum_{z\,:\,q(z)\,=\,w} P_z X P_z,$$

and two recorded residual channels

$$\begin{aligned} \mathcal{F}(X) &= \textstyle\sum_w \mathrm{ketbra}\, ww_R \otimes T(\Delta_w(X)), \\ \mathcal{G}(X) &= \textstyle\sum_w \mathrm{ketbra}\, ww_R \otimes \mathcal{I}_w(T(X)), \end{aligned}$$

where $\{\mathcal{I}_w\}_w$ is an instrument on K.

Proposition (Instrument-level observation criterion). *The classical record laws of $\mathcal{F}$ and $\mathcal{G}$ agree for every input state if and only if*

$$T^*(E_w) = \sum_{z\,:\,q(z)\,=\,w} P_z \qquad \text{for every } w.$$

For a declared residual algebra $\mathcal{O} \subseteq L(K)$, the channels agree on all recorded residual observables $\sum_w \mathrm{ketbra}\, ww_R \otimes B_w$, $B_w \in \mathcal{O}$, if and only if

$$\sum_{z\,:\,q(z)\,=\,w} P_z T^*(B) P_z = T^*(\mathcal{I}_w^*(B)) \quad (w,\, B \in \mathcal{O}).$$

Both are operator identities and therefore remain valid under an untouched reference extension.

Proof. The fine recorded probability of w is

$$\mathrm{Tr}\,(T(\Delta_w(\rho))) = \mathrm{Tr}\left(\rho \sum_{z\,:\,q(z)\,=\,w} P_z\right).$$

The coarse probability is $\mathrm{Tr}\,(\rho T^*(E_w))$. Equality for every state is equality of their Hermitian effects, proving .

Pairing the w-block with B gives the two pullbacks in . Equality against every state, first for self-adjoint B and

then by linearity, is equivalent to their operator equality. Tensoring these linear-map identities with a reference identity preserves them.

Theorem (Residual obstruction with an identity coarse instrument). *Record agreement does not imply residual-source agreement, even when* $T = \mathrm{id}$ *and the coarse instrument has one outcome with map* $\mathscr{I}_* = \mathrm{id}$*. There is a qubit input for which the recorded residual outputs have trace distance* $1/2$*.*

Proof. Take the computational projectors P_0, P_1, aggregate both outcomes to *, and take $T = \mathrm{id}$. Both record effects are I, so the record laws agree on every input.

For $\rho = \mathrm{ketbra}\,{+}{+}$,

$$\mathscr{G}(\rho) = \mathrm{ketbra}\,{*}{*}_R \otimes \mathrm{ketbra}\,{+}{+}, \qquad \mathscr{F}(\rho) = \mathrm{ketbra}\,{*}{*}_R \otimes I_2/2.$$

Their difference has residual block $X/2$, where $X = \mathrm{ketbra}\,01 + \mathrm{ketbra}\,10$. Its eigenvalues are $1/2, -1/2$. Hence

$$\|\mathscr{G}(\rho) - \mathscr{F}(\rho)\|_1 = 1, \qquad d_{\mathrm{tr}}(\mathscr{G}(\rho), \mathscr{F}(\rho)) = \frac{1}{2}.$$

The residual observable X has expectations 1 and 0, respectively.

Here the *coarse instrument* is the identity. The aggregated *fine residual channel* is dephasing and is not the identity. Confusing these two maps would invalidate the example. This is the instrument-level distinction emphasized in [14].

Corollary (Robust residual distinction). *If implementations of the two outputs in have trace-distance errors at most* ε_{f} *and* ε_{c}*, their mutual trace distance is at least*

$$\frac{1}{2} - \varepsilon_{\mathrm{f}} - \varepsilon_{\mathrm{c}}.$$

Proof. Apply the triangle inequality through the two ideal outputs.

A record current and a residual current are different named interfaces. Neither is, merely by that name, a stress tensor.

Configurations and complete control

Definition (Configuration). A configuration is

$$C = (g; \Gamma; \rho; \ell; h; v_G, v_{\mathrm{src}}; \mathfrak{r}),$$

where g is the complete geometric description, Γ, ρ are the named matter interface and state, $\ell : \Gamma \to \mathrm{Cell}(g)$ is the abstract location map, h is finite controller data, v_G, v_{src} are separate permission registers, and $\mathfrak{r}$ is the resource reservation. A certified g includes its accepted certificate.

A location label is not a material trajectory, a mass assignment, or a stress tensor.

Assumption (Deterministic spectator-blind specification). For a fixed source embedding, the complete update specification is a deterministic function of

$$g, \quad h, \quad v_G, v_{\mathrm{src}}, \quad \mathfrak{r}_{\mathrm{geom}}, \quad \omega_\rho|_{\mathcal{A}_{\mathrm{src}}},$$

and any additional explicitly declared accessible inputs.

Equal such inputs select the same rule, guard result, support, proposed geometry, evaluated packet, certificate presentation, successor controller state, and transport of original names. They also select the same declared matter channel whenever a matter channel is included.

Every used record, seed, program counter, permission bit, resource test, and observable is part of that declared specification. Undeclared state descriptions and remote scheduler inputs are excluded.

Adding inaccessible spectators leaves the original label dictionary and all original input interpretations unchanged. Matter maps extend by the spectator identity, and accessible observables extend by $B \mapsto B \otimes I_N$. Geometric and coupling reservations are independent of spectator number. Spectator transport and history storage are reserved separately; their outcomes and capacity tests are not additional source inputs.

This concerns the entire specification, not only one displayed source expectation. It is the local control discipline needed for the comparisons with [14, 15].

An exact incidence clock and a rounded length report are different inputs. A *G*-guard uses its declared exact clock. It may not substitute a zero rounded report or read occupancy without changing the interface.

Option N and the recovery boundary

We retain the deterministic nonlinear option, Option N:

$$g' = F(g, \omega_{\rho|_{\mathcal{A}_{\mathrm{src}}}}),$$

with the other declared arguments suppressed. The dependence need not be affine in ρ. Consequently, the complete matter–geometry transformation is not asserted to be a CPTP map.

The source functional is a *semantic input* to this model. No single-copy protocol for acquiring exact expectations, no finite-gate implementation of an arbitrary expectation law, and no physical nonlinear feedback mechanism is proved. An implementation based on a supplied promised token would be a separate interface. A quantitative companion to that semantic-input discipline is recorded on attached typographic parent VIII-ST Version 1 as the finite-promise discrimination bound $\max\{e_m, e_n\} \geq \frac{1}{2} - |z_m - z_n|/4$ (prop:finite-promise) [20]. That bound is a source record, not a theorem of the present sheet.

The controller comparisons below are pairwise equality statements at aligned steps. They are not a general no-signalling theorem for arbitrary nonlinear feedback.

Likewise, informational determination is not a CPTP decoder. For a linear channel on a complete logical input space, exact recovery requires a constant *complete* complement; states and proves that criterion and an injective-channel counterexample. The criterion is not a subsystem-recovery criterion [4, 21]. A certified geometric output therefore cannot be read as recovery of an unknown quantum input.

Spinless affine geometry and realization

Affine conventions

Use coordinates $z = (T, X, Y)$ with

$$\eta = \operatorname{diag}(-1, 1, 1).$$

Affine isometries are written $(L, b) \in \mathrm{ISO}^{+}(2,1)$, with $L \in \mathrm{SO}^{+}(2,1)$, and act by

$$(L, b)z = Lz + b, \qquad (L, b)(L', b') = (LL', Lb' + b).$$

Thus

$$(L, b)^{-1} = (L^{-1}, -L^{-1}b).$$

For $0 < \mu < 2\pi$, define

$$R(\mu) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\mu & -\sin\mu \\ 0 & \sin\mu & \cos\mu \end{pmatrix}, \qquad J(\mu) = (R(\mu), 0).$$

Let $e_0 = (1,0,0)^{\mathrm{T}}$ and $\mathscr{W}_0 = \mathbb{R}e_0$.

For a reference affine frame $A = (C, a)$, put

$$\begin{aligned} u &= Ce_0, \qquad \mathscr{W} = a + \mathbb{R}u, \\ H_{\Pi} &= AJ(\mu)A^{-1} \\ &= \big(CR(\mu)C^{-1}, a - CR(\mu)C^{-1}a\big). \end{aligned}$$

The vector u is future unit timelike.

Definition (Spinless packet). A spinless packet is

$$\Pi = (\mathrm{p}, \mu, \mathscr{W}, u, A, H_{\Pi}),$$

where p is a geometric particle identifier and the other entries satisfy . The orientation and the interval $0 < \mu < 2\pi$ are part of the data.

Lemma (Spinless affine matching). *The map H_{Π} fixes $\mathscr{W}$ pointwise, its Lorentz part fixes u, and its oriented rotation on $u^{\perp}$ has angle μ. These conditions determine H_{Π} uniquely from $\mu, \mathscr{W}, u$ and the declared orientation.*

Proof. For $z = A(te_0)$,

$$H_{\Pi}z = AJ(\mu)(te_0) = A(te_0) = z.$$

The Lorentz part fixes Ce_0 and acts on its oriented orthogonal plane by the conjugate spatial rotation.

Conversely, a proper future-preserving Lorentz map fixing u restricts to an orientation-preserving Euclidean rotation on $u^{\perp}$. The prescribed oriented angle determines its Lorentz part L. Pointwise fixation of $a + \mathbb{R}u$ then requires $b = (I - L)a$. Replacing a by $a + t_0 u$ does not change this translation.

A trace alone does not determine the oriented angle. A coherent change of common affine framing conjugates all holonomy data and transports the worldline and rest vector.

The cone dictionary

Let $L_\mu = 2\pi - \mu$. The spatial cone C_μ is the wedge $0 \le \theta \le L_\mu$, with the two boundary rays identified and the origin completed to an apex. The regular metric is

$$\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2.$$

The spacetime is

$$M_\mu = \mathbb{R}_T \times C_\mu, \qquad \mathrm{d}s^2 = -\mathrm{d}T^2 + \mathrm{d}r^2 + r^2 \mathrm{d}\theta^2.$$

The upper boundary ray is mapped to the lower ray by $J(\mu)$, because

$$\mathrm{Rot}\,(\mu)\begin{pmatrix} \cos L_\mu \\ \sin L_\mu \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

This is wedge gluing, not a quotient of the entire plane by the group generated by a rotation. We take this spinless conical dictionary as the admissibility model. A normalization such as $\mu = 8\pi G m$ would be additional dictionary input. The singular particle interpretation is not derived from an observation-boundary identity.

Regular charts are Minkowski charts, so their curvature and Einstein tensor vanish. The singular interpretation is the supplied conical dictionary, consistent with the classical setting of [1, 2].

Realization before matching

Definition (Realized description). A realized polygonal description supplies:

1. a marked ambient cone, its orientation, reference frame, worldline, and spinless packet;

2. a finite polygonal patch and a prescribed exterior;

3. developed face polygons, their affine planes, labelled vertices and edges, and declared overlap components;

4. a seam-compatible embedded slice, spacelike on each regular face, together with the claimed global acausality verification when used;

5. positive active edge lengths and positive face areas on each regular parameter epoch;

6. a marked collar and common based-loop framing;

7. for an event, two one-sided realized families with a common limiting slice and explicitly identified collapsed slots. An unbounded exterior may have finitely many cell labels. Collapsed event slots are metadata, not zero-dimensional replacements for active edges or faces.

A one-sided incidence germ here means the restriction of a realized family to an open parameter interval adjacent to the event, together with its labelled limiting data. Positivity is required on that open epoch, not for a collapsed outgoing face at the event itself.

For each face f, let $\mathcal{A}_f \in \mathrm{ISO}^+(2,1)$ be its adapted chart change from the appropriate cut-space development. For an oriented overlap component from f to g, let $n_{gf} \in \{-1, 0, 1\}$ denote its signed seam crossing. Chart paths retain their individual overlap components. The prescribed transition is

$$t^{\mathrm{can}}_{g \leftarrow f} = \mathcal{A}_g J(\mu)^{n_{gf}} \mathcal{A}_f^{-1}.$$

Holonomy alone does not determine these individual maps: the realization, adapted charts, and specific overlap component are also required.

The finite admissibility contract

The five matching components

Definition (Matching packet and certificate). For a realized description and proposed transition maps, the evaluated matching packet is

$$\mathrm{Ein}_0(g) = (\mathscr{V}(g), \mathscr{R}(g), \mathscr{H}(g), \mathscr{K}(g), \mathscr{W}(g)).$$

It records the following checks:

1. Vertex and overlap matching. Proposed maps equal on the specified components; reverse maps are inverses; paired endpoints and segments match; developed polygonal boundaries close; and regular local vertex words are affine identities.

2. Particle matching. The proposed peripheral map equals H_{Π} in the declared framing. The deficit, affine worldline, and future rest direction satisfy .

3. Marked holonomy words. Ordered chart words, overlap components, connecting paths, and their topological classes are supplied. A positive primitive particle meridian has winding one, independently of its matrix value.

4. Collar matching. Core and collar share their prescribed boundary realization and transported reference framing. Their primitive peripheral words equal the same framed H_{Π}.

5. Worldline data. Every chart describes the transported singular worldline and rest vector consistently. The peripheral map fixes the worldline pointwise, and the declared slice intersection belongs to that worldline. A complete certificate Cert(g) contains finite witness tables, exact equalities, winding evidence, and realization verification. It is accepted only when every applicable geometric and matching obligation holds.

Developed polygonal boundary closure means closure of the supplied boundary segments in the appropriate development. It is not an instruction to impose identity holonomy on a particle meridian.

Remark (Meaning of completeness). Completeness is relative to this declared affine-conical contract. It does not classify all 2 + 1 initial data, prove membership in every canonical polygon gauge, or turn an arbitrary matrix list into an embedded slice. A certificate is not an unconstrained string saying "pass," and no executable

exact-arithmetic checker is constructed in this paper.

Persistent signature

An event changes the numbers of faces, vertices, and matching equations. Literal equality of the two raw Ein_0 lists is therefore not required.

For a commonly marked punctured-disk domain, define

$$\mathrm{Sig}(g) = (\mathrm{p}, \mu, \mathscr{W}, u, \text{reference framing}, [\gamma]_{\text{primitive}}, H_{\Pi}, \text{collar agreement}).$$

The slice-intersection point is checked in (E5), but it is not required to remain the same point of the fixed worldline as the slice advances.

Definition (Constraint-preserving geometric update). A geometric update is constraint preserving if it:

1. returns an accepted output certificate, or a certified outgoing germ with its common event limit;

2. preserves Sig under the declared marking;

3. supplies the required original-name location transport. The realization-dependent equations are recomputed.

Four rule types

Interface contract (Complete typed outputs). Every rule returns its complete typed configuration and the applicable certificate.

1. A ***G*-rule** reads only declared geometric, geometric-controller, permission, and resource inputs. It preserves Sig and does not inspect matter state, occupancy, or spectator cardinality.

2. A **matter rule** applies a declared channel on its named interface, extended by identities on untouched factors. It does not change geometry.

3. An **ordinary unsourced coupling rule** may read a named source functional but leaves the particle signature fixed. A packet-changing rule is not of this type.

4. A **sourced coupling rule** proposes a new packet

$$\Pi' = F_{\Pi}(\Pi, \omega_{\rho}|_{\mathcal{A}_{\mathrm{src}}})$$

and must also return its complete realization, charts, gluing data, collar, transport, and output certificate. The packet proposal alone is insufficient. Geometric and coupling parts leave internal matter unchanged. Any included matter channel is separately declared and obeys the same untouched-factor requirements.

This contract states what a sourced constructor must return; it does not construct one.

Proposition (Affected-output reevaluation). *If an update changes an argument of an obligation in (E1)–(E5), that obligation and all other affected checks must be reevaluated on the output. An incompatible input certificate does not certify the new data.*

Proof. Acceptance verifies evaluated output equalities and output realization requirements. An input equality with changed arguments is not the required output equality. If incompatible old values are retained, the corresponding output equality fails. Unaffected witnesses may be reused only when they still verify the same obligation.

Completeness and observation invariance

A1: uniqueness in the fixed presentation

Theorem (Fixed-presentation completeness). *Fix a realized description, its adapted charts, packet, reference framing, overlap components, and seam maps. Two proposals agreeing on these data but differing on one prescribed transition cannot both be certified. The same conclusion holds if several transitions differ.*

Proof. The fixed data prescribe the unique affine map

$$\mathcal{A}_g J(\mu)^{n_{gf}} \mathcal{A}_f^{-1}.$$

Condition (E1) requires the proposed transition to equal this map. Two different affine maps cannot both equal it. Apply this argument to any differing component.

Example (Why the fixed presentation matters). If a chart transition is $t = \mathcal{A}_g \mathcal{A}_f^{-1}$, replacing $\mathcal{A}_g$ by $B_g \mathcal{A}_g$ gives $B_g t$. Closed words undergo the corresponding base-chart conjugation, and the geometric structure may be

unchanged. Thus different valid chart presentations can have different transition maps. Peripheral holonomy also does not determine edge lengths or face shapes.

A2: factorization through accessible inputs

Theorem (Constraint-blindness). *Let two configurations have the same geometry, controller state, permissions, geometric resource reservation, source embedding, and all other declared accessible inputs. Suppose*

$$\omega_{\rho}|_{\mathcal{A}_{\mathrm{src}}} = \omega_{\sigma}|_{\mathcal{A}_{\mathrm{src}}}.$$

Under , their evaluated input packets agree. They receive the same applicability decision, proposed update, output packet, and deterministically selected certificate. Either both are certified or both are rejected.

Proof. The input packet is determined by the common geometric data. Every argument of the deterministic specification agrees. Its guard, geometry, packet, support, transport, certificate, and successor controller outputs therefore agree. The evaluated acceptance conditions have the same truth value.

This does not choose F_{Π} or prove existence of a nonconstant admissible sourced realization.

Example (Permissions are not erased by source equality). A rule with guard $\nu_{\mathrm{src}} \geq 1$ can have different applicability on permissions 0 and 1, even when geometry and source functionals agree. Equal permissions are therefore an essential hypothesis.

Arbitrary spectator extensions

Lemma (Restriction of a linear matter map). *For any linear map $\Psi : L(\mathscr{H}_{\Gamma}) \to L(\mathscr{H}_{\Gamma'})$ and any operator X on ΓN,*

$$\mathrm{Tr}_N[(\Psi \otimes \mathrm{id}_N)(X)] = \Psi(\mathrm{Tr}_N X).$$

No product-state or trace-preservation assumption is needed.

Proof. Pair both sides with any output operator B. Both pairings are

$$\mathrm{Tr}\,((\Psi^{*}(B) \otimes I_N)X).$$

Equality against all B proves the identity.

Theorem (Spectator-blind certified executions). *Consider a finite execution of the rule types in . Extend its initial state to an admissible joint state $\hat{\rho}$ on ΓN such that*

$$\mathrm{Tr}_N \hat{\rho} = \rho.$$

Assume the complete blindness and extension clauses of , identical original locations, and legal separate spectator transport/storage.

Then an aligned extended execution is legal. At every step, the original reduced matter states, geometric descriptions, persistent signatures, certification decisions, original transports, and declared controller outputs agree.

Proof. Induct on steps. For each accessible observable,

$$\mathrm{Tr}\,((B \otimes I_N)\hat{\rho}) = \mathrm{Tr}\,(B\rho).$$

Equal complete inputs therefore select the same rule, guard, support, and certificate.

A matter channel preserves the reduction identity by . A G-rule receives the same geometric/controller inputs and leaves internal matter unchanged. A coupling rule receives the same accessible functional, so gives the same geometric and certification output. Any common included matter channel again preserves the original reduction. Original transport and successor control agree. The separate-reservation clauses establish spectator legality without changing these decisions.

The extension may be entangled and need not be a fixed pure or product spectator. The theorem identifies the original reductions, not arbitrary full joint extensions.

A guard $|\Gamma| \le K$ would violate the hypotheses: one product spectator could change its decision without changing any accessible source expectation.

Remark (Selective comparisons). Equation also applies to an unnormalized completely positive instrument

branch. For paired branches selected from equal declared inputs, the original unnormalized reductions and branch probabilities agree. Conditional reductions agree on positive-probability branches. This is a branchwise comparison, not a claim that the spectator conditional state remains unchanged, and it does not promote Option N to a joint CPTP model.

Restricted immediate versus deferred readout

Lemma (Common destructive observation). *The isometry V in satisfies*

$$\mathrm{Tr}_S(VXV^\dagger) = \sum_z \mathrm{ketbra}\, zz_A \otimes M_z X M_z^\dagger.$$

Let Λ be a channel acting only on B, and let $\mathcal{N}_A$ destructively read A into R. Then, with the required identity extensions,

$$\mathrm{Tr}_S \mathcal{N}_A((\mathrm{id}_{SA} \otimes \Lambda)(VXV^\dagger)) = (\mathrm{id}_R \otimes \Lambda)\mathcal{M}_S(X).$$

The identities include every untouched external reference.

Proof. Expand $X = \sum_{z,z'} \mathrm{ketbra}\, zz'_S \otimes X_{zz'}$. Coherent recording appends $\mathrm{ketbra}\, zz'_A$. Tracing *S* removes exactly the terms $z \neq z'$, giving the first formula. Operations on *B* act on the blocks $X_{zz'}$ and commute with the indicated restriction and carrier readout. This proves . A reference is another untouched factor in the same calculation.

Corollary (Constraint compatibility on the deferred boundary). *Compare an immediate destructive-readout execution with a coherent-recording execution. During their continuations suppose:*

1. *source and control inputs use only the evolving complement B, not R, A, or retained S;*

2. *initial geometry, controller data, permissions, resource inputs, and original complement locations agree;*

3. *all continuation matter channels act only on B, are selected equally from equal accessible inputs, and extend by identities on excluded factors;*

4. *excluded-factor storage and transport are separately legal and cannot affect guards or geometric decisions.*

Their geometric outputs and certification decisions agree at every aligned continuation step. After terminal

readout of A, the record–complement outputs agree when the deferred output is additionally restricted away from S.

Proof. Initially, both *B*-reductions are $\mathrm{Tr}_S \rho$. Equal complement reductions give equal accessible source functionals. Constraint-blindness selects the same geometric and control outputs, and the common complement channel preserves equality by . Induction proves the continuation assertion.

For the fixed initial pair, these common choices select the same finite sequence of complement channels. Their composition is a common channel Λ. Apply to Λ to obtain the terminal record–complement equality.

The terminal carrier *A* has been consumed. The additional retained factor to be removed is *S*, not *A*. If the immediate controller reads *R* during continuation, this corollary does not apply. A record-dependent comparison requires different hypotheses.

Holonomy on the marked punctured disk

Ordered products and based transport

A chronological chart path is $f_0, f_1, \dots, f_m$, with each overlap component specified. The last transition acts on the left:

$$t_{f_m \leftarrow f_{m-1}} \cdots t_{f_1 \leftarrow f_0}.$$

The based-loop package supplies a common collar basepoint and affine frame, a chosen connecting path to each loop, its transport, and a marked primitive positive peripheral class. A connecting path is data, not a topologically unique path.

If Q transports the common reference coordinates into the starting chart and W_f is the chart-based loop word, then

$$W_{\mathrm{ref}} = Q^{-1} W_f Q.$$

Peripheral equality always means equality after transport to the same basepoint/frame.

Lemma (Canonical word formula). *For a closed canonical chart path,*

$$\mathrm{Hol}(\gamma) = \mathcal{A}_{f_0} J(\mu)^{N(\gamma)} \mathcal{A}_{f_0}^{-1}, \qquad N(\gamma) = \sum_{k=0}^{m-1} n_{f_{k+1} f_k}.$$

On the marked punctured cone slice, $N(\gamma)$ is its oriented winding number.

Proof. Substitute . Adjacent adapted-chart factors cancel, leaving the displayed power of $J(\mu)$.

Lift angular position to the cover with period $L_\mu = 2\pi - \mu$. A positive crossing of the marked seam increases the sheet index by one; a negative crossing decreases it by one. A closed path has angular increment $N(\gamma)L_\mu$, which is its winding number times the angular period.

Example (Written seam crossings can cancel). A small regular loop crossing the cut once positively and once negatively has $N = 1 - 1 = 0$, hence identity affine product. A change of written charts does not make that loop a particle meridian.

Example (Identity holonomy need not mean contractibility). At $\mu = \pi/2$, $J(\pi/2)^4 = I$. A fourfold positive particle loop has winding four and is noncontractible, despite its identity affine matrix. The matrix representation is not faithful to the integer winding group.

A finite presentation

Definition (Declared edge-path presentation). A finite presentation consists of a connected finite cell structure K, a marking identifying its based edge-path homotopy group with the fundamental group of the punctured-disk domain, and:

1. invertible transition values on oriented edges, with inverse values on reversed edges;

2. regular two-cell boundary relators;

3. a marked primitive positive peripheral edge word;

4. the frame transports used to compare based words. One may use a triangulated compact annular truncation and a finite refinement with all required boundary paths. A dual structure is allowed only when its marking actually gives this presentation.

Lemma (Local relators and one generator suffice). *In a presentation satisfying , suppose every regular two-cell boundary has identity ordered product and the marked primitive positive peripheral word has product H in the common frame. Then every based loop of winding w has product H^w. Conversely, this loop formula implies those local and peripheral conditions.*

Proof. Deleting an edge followed by its reverse preserves the product. The identity boundary relators permit replacement of one boundary path of a regular two-cell by its complementary path without changing the product. The declared presentation makes these moves the required edge-path homotopies.

The punctured disk, or its annular truncation, has fundamental group $\mathbb{Z}$, generated by the marked positive meridian. A winding-w loop is based-homotopic to its w-fold power, with negative powers represented by reversed loops. Product invariance gives H^w.

Conversely, regular boundaries have winding zero and the marked meridian has winding one.

The peripheral word is a generator with prescribed affine value, not an identity relator filling the puncture. Small vertex loops give regular relators only within a declared presentation that implements the required topology.

A3: matching reduction after realization

Theorem (Holonomy reduction). *Fix a realized marked spinless cone patch with punctured-disk topology, its packet, adapted charts, overlap components, based-loop package, and a presentation as in . After checking endpoints, developed face closure, reverse maps, and prescribed transitions in (E1), the holonomy obligations are equivalent to:*

1. *identity products at regular relators;*

2. *one primitive positive peripheral product equal to H_{Π};*

3. *a collar primitive product equal to the same framed H_{Π}. Other based words follow from winding and transport. Particle/worldline matching additionally requires the supplied worldline, rest-vector, and slice-intersection data and the algebraic conditions of .*

These word checks do not replace realization, spacelikeness, positivity, primitive-winding evidence, or admission to the spinless dictionary.

Proof. The finite-relator lemma gives every based product from (a) and (b). Condition (c) verifies the particular collar identification in the common frame. The spinless lemma determines the required pointwise-axis map from the supplied packet. Chart transport verifies the representations of the worldline and rest vector; the slice intersection is separately checked.

Conversely, the complete packet includes the specified regular, peripheral, and collar words. None of these matrix products controls a face gradient or proves embedding, so realization remains an independent prerequisite.

The complete fixed-cone construction

The formulas in this section independently reproduce the fixed slicing compared with VII-F [17]. No predecessor theorem is needed for their verification.

Marked sectors and the graph family

Set

$$\mu_0 = \frac{\pi}{2}, \qquad L_{\mu_0} = \frac{3\pi}{2}, \qquad J_0 = J(\pi/2), \qquad \mathscr{W} = \mathscr{W}_0.$$

Let

$$d_i = \begin{pmatrix} \cos(i\pi/2) \\ \sin(i\pi/2) \end{pmatrix}, \qquad i = 0, 1, 2, 3.$$

Sector i, $i = 0, 1, 2$, has orthonormal coordinates

$$x = \xi d_i + \zeta d_{i+1}, \qquad \xi, \zeta \geq 0.$$

Cone labels are cyclic modulo three. Developed vectors $d_0, \ldots, d_3$ are not cyclically identified until the seam is glued.

In particular, a vertex with cone label A_0 has lower development $3d_0$ and upper development $3d_3$. Whenever a

polygon in sector two uses A_{i+1}, B_{i+1}, or D_{i+1}, it uses the upper developed copy. The seam rotation sends it to the corresponding lower copy. This convention applies to every table below.

Define the cone functions sectorwise by

$$\rho(x) = \xi + \zeta, \qquad h(x) = \frac{2}{3}(\xi + \zeta), \qquad k(x) = \frac{3}{5}\min\{\xi, \zeta\}.$$

On each shared ray,

$$\rho = r, \qquad h = \frac{2r}{3}, \qquad k = 0.$$

They therefore descend to the cone.

For $-1 \le s \le 1$, let

$$F_s(x) = s + \min\{2, \max\{h(x), s + k(x)\}\}, \qquad \Sigma_s = \{(F_s(x), x) : x \in C_{\mu_0}\}.$$

Lemma (Uniform realization bounds). *The graphs Σ_s are seam compatible, embedded, and globally acausal. Each regular face is uniformly spacelike, with*

$$\lambda = \frac{2\sqrt{2}}{3} < 1, \qquad G_{\text{face}} \ge \frac{1}{9} I_2.$$

For $s_2 > s_1$,

$$s_2 - s_1 \le F_{s_2}(x) - F_{s_1}(x) \le 2(s_2 - s_1).$$

Moreover,

$$h - k \ge \frac{11}{30}\rho, \qquad F_s = 2 + s \quad \text{when} \rho \ge 3.$$

The family has a common uniform limit at $s = 0$.

Proof. The nonzero affine branch gradients in a sector are

$$(2/3, 2/3), \qquad (3/5, 0), \qquad (0, 3/5).$$

Their Euclidean norms are bounded by λ. Minimum and maximum of real functions with a common Lipschitz

constant retain that constant. Shared-ray values give continuity across all rays and the seam.

Integrating the sectorwise bound along spatial paths, including paths through shared rays or the apex, gives

$$|F_s(x) - F_s(y)| \le \lambda d_{C_{\mu_0}}(x, y).$$

Graph projection is a homeomorphism onto the cone, so the graph is embedded. On a face with gradient a, the induced metric is $I_2 - aa^{\mathrm{T}}$, and

$$v^{\mathrm{T}}(I_2 - aa^{\mathrm{T}})v \ge (1 - \lambda^2)|v|^2 = \frac{1}{9}|v|^2.$$

A causal curve in the static product has elapsed absolute time at least the length of its spatial projection. Distinct graph points have smaller absolute time separation than their spatial distance, because $\lambda < 1$. They cannot be causally related.

For fixed x, the capped inner expression is nondecreasing and 1-Lipschitz in s. Adding s gives . Since $\min\{\xi, \zeta\} \le \rho/2$,

$$h - k \ge (2/3 - 3/10)\rho = \frac{11}{30}\rho.$$

If $\rho \ge 3$, then $h \ge 2$, so the outer minimum equals 2. The upper bound in also gives uniform convergence to F_0 as $s \to 0$.

No regular tangent plane at the singular particle is asserted. The worldline intersection is

$$P_s = \begin{cases} (s, 0, 0), & s \le 0, \\ (2s, 0, 0), & s \ge 0. \end{cases}$$

It moves along the fixed worldline; this does not change the packet.

Take the compact patch $\rho \le 4$ and the marked collar $7/2 \le \rho \le 4$. The collar and exterior are resting spatially, with equation $T = 2 + s$. "Prescribed exterior" means this translating resting exterior, not a frozen embedded time slice.

The exact cells

Write $A_i = 3d_i$ and $D_i = 4d_i$ for spatial vertices, with times $T = 2 + s$. For $s \le 0$, the core faces are

$$V_i = [P, A_i, A_{i+1}], \qquad T = s + \frac{2}{3}(\xi + \zeta).$$

For $s > 0$, introduce spatial vertices

$$B_i = \frac{3}{2} s\, d_i, \qquad C_i = \frac{15}{11} s(d_i + d_{i+1}),$$

and use

$$\begin{aligned} U_i^- &= [P, B_i, C_i], & T &= 2s + \tfrac{3}{5}\zeta, \\ U_i^+ &= [P, C_i, B_{i+1}], & T &= 2s + \tfrac{3}{5}\xi, \\ V_i &= [B_i, A_i, A_{i+1}, B_{i+1}, C_i], & T &= s + \tfrac{2}{3}(\xi + \zeta). \end{aligned}$$

Their new-vertex times are

$$T(B_i) = 2s, \qquad T(C_i) = \frac{31}{11} s.$$

The collar-side faces are

$$Q_i = [A_i, D_i, D_{i+1}, A_{i+1}], \qquad T = 2 + s.$$

Outside the patch, retain three horizontal unbounded sector faces and the persistent ray subdivisions.

Lemma (Dominance regions give the cell decomposition). *The listed polygons cover the graph and have disjoint face interiors. The cellulation is constant on each open epoch $s < 0$ and $s > 0$. For $s > 0$, the new cells lie strictly inside $\rho = 3$ and are not cut by the horizontal cap.*

Proof. For $s \le 0$, the bound $k \le h$ gives $s + k \le h$. The core is therefore the three triangles V_i.

For $s > 0$, in the half-sector $\xi \ge \zeta$,

$$h = s + k \quad \Leftrightarrow \quad 10\xi + \zeta = 15s.$$

This segment joins B_i to C_i. In the other half-sector,

$$h = s + k \quad \Leftrightarrow \quad \xi + 10\zeta = 15s,$$

joining C_i to B_{i+1}. The two triangles are precisely the regions where $s + k$ dominates; their complement in the outer

triangle is the listed simple pentagon.

At the new vertices,

$$\rho(B_i) = \frac{3}{2}s, \qquad \rho(C_i) = \frac{30}{11}s \le \frac{30}{11} < 3.$$

Thus the new cells remain away from the crest. On them,

$$s + k \le s + k(C_i) = \frac{20}{11}s < 2,$$

so the cap does not remove a new cell. Outside $\rho = 3$, gives the horizontal faces. These exact dominance regions have disjoint interiors and exhaust the graph.

Lengths, areas, and nondegeneracy

Let e_i be the persistent abstract edge label, realized as $[P, A_i]$ incoming and $[B_i, A_i]$ outgoing. For finite edges, direct Minkowski lengths give

$c\|c\|c$edge	$s < 0$	$s > 0$
hline e_i	$\sqrt{5}$	$\sqrt{5}(1 - s/2)$
$[A_i, A_{i+1}]$	$3\sqrt{2}$	$3\sqrt{2}$
$[A_i, D_i]$	1	1
$[D_i, D_{i+1}]$	$4\sqrt{2}$	$4\sqrt{2}$
$[P, B_i]$	absent	$3s/2$
$[P, C_i]$	absent	$3\sqrt{41}\, s/11$
$[B_i, C_i]$, $[C_i, B_{i+1}]$	absent	$3\sqrt{65}\, s/22$

The induced face areas are

$$\begin{aligned} \operatorname{Area}(V_i) &= \begin{cases} 3/2, & s \le 0, \\ 3/2 - 15s^2/22, & s > 0, \end{cases} \\ \operatorname{Area}(U_i^{\pm}) &= \frac{9s^2}{11} \quad (s > 0), \\ \operatorname{Area}(Q_i) &= \frac{7}{2}. \end{aligned}$$

Lemma (Exact metric tables). *Equations are the intrinsic edge lengths and areas of the listed faces. All active finite cells are nondegenerate on their open epochs.*

Proof. For each segment,

$$\ell^2 = |\Delta x|^2 - (\Delta T)^2.$$

Incoming, PA_i has spatial displacement 3 and time displacement 2, giving $\sqrt{5}$. Outgoing,

$$\ell(e_i)^2 = (3 - 3s/2)^2 - (2 - s)^2 = 5(1 - s/2)^2.$$

The horizontal edges give their displayed Euclidean lengths. The edge PB_i has zero time displacement.

For PC_i,

$$\ell(PC_i)^2 = 2(15s/11)^2 - (9s/11)^2 = \frac{369s^2}{121}.$$

For B_iC_i, in sector coordinates,

$$(\Delta T, \Delta\xi, \Delta\zeta) = \frac{s}{22}(18, -3, 30),$$

so

$$\ell(B_iC_i)^2 = \frac{s^2}{22^2}(9 + 900 - 324) = \frac{585s^2}{484}.$$

Reflection gives the same length for C_iB_{i+1}.

The projected outer triangle has area 9/2. Each new projected triangle has area

$$\frac{1}{2}|\det\begin{pmatrix} 3s/2 & 15s/11 \\ 0 & 15s/11 \end{pmatrix}| = \frac{45s^2}{44}.$$

On a plane $T = c + a \cdot x$, area density is $\sqrt{1 - |a|^2}$. It is 1/3 on V_i and 4/5 on $U_i^{\pm}$. Thus

$$\text{Area}\,(V_i) = \frac{1}{3}\left(\frac{9}{2} - \frac{45s^2}{22}\right) = \frac{3}{2} - \frac{15s^2}{22},$$

and

$$\text{Area}\,(U_i^{\pm}) = \frac{4}{5}\frac{45s^2}{44} = \frac{9s^2}{11}.$$

The horizontal quadrilateral has area $(4^2 - 3^2)/2 = 7/2$.

Every new length is a positive coefficient times s, and every new area is a positive coefficient times s^2. For $0 < s \le 1$,

$$\ell(e_i) \ge \frac{\sqrt{5}}{2}, \qquad \operatorname{Area}(V_i) \ge \frac{9}{11}.$$

All remaining finite lengths and areas are constant positive values. This proves nondegeneracy.

The unique isolated event

Theorem (Unique particle-incidence event). *The family has exactly one incidence-event parameter in* $[-1, 1]$, *namely* $s = 0$. *At that parameter:*

1. *the six vertices* B_i, C_i *coalesce at* P_0*;*

2. *the twelve new edges* $PB_i, PC_i, B_iC_i, C_iB_{i+1}$ *collapse;*

3. *the six faces* $U_i^{\pm}$ *collapse;*

4. *the particle link changes from three incident faces to six. There is no other collision, vanishing active finite edge or face, or contact of a new cell with the outer crest. At* $s = 0$, *the regular cellulation is the common limiting three-face core; outgoing collapsed slots are germ metadata.*

Proof. The exact dominance regions show constant incidence on each sign-preserving open epoch. For $s > 0$, the explicit new coordinates are distinct, and gives positive active lengths and areas. The bound $\rho(C_i) \le 30/11 < 3$ excludes crest contact. Coverage and nonintersection were established by . The incoming finite cells are nondegenerate and have unchanged incidence.

All new coordinates are proportional to s, so their specified cells collapse only at $s = 0$. The persistent e_i, V_i have the incoming nondegenerate segments and triangles as their limits.

Incoming, the particle meets exactly the three V_i. Outgoing, it meets exactly the six $U_i^{\pm}$; the pentagons no longer touch it. Across each bisector edge PC_i, the gradients are $(3/5, 0)$ and $(0, 3/5)$. Across a ray edge PB_i, unfolding the adjacent sectors changes the sign of the transverse gradient. These are genuine creases. The six-face link is

therefore not a mere subdivision of one plane or a relabelling of an unchanged star.

As a combinatorial check, the compact patch counts are

$c\|ccc\|c$	vertices	edges	faces	$V-E+F$
hline $s<0$	7	12	6	1
$s>0$	13	24	12	1.

The count supports the explicit disk cellulation; it is not a substitute for its realization proof.

Adapted affine charts

For $|g|<1$, define

$$\mathscr{B}(g)=\begin{pmatrix}\gamma & -\gamma g^{\mathrm{T}}\\ -\gamma g & I_2+(\gamma-1)gg^{\mathrm{T}}/|g|^2\end{pmatrix},\qquad \gamma=(1-|g|^2)^{-1/2},$$

with $\mathscr{B}(0)=I_3$. It sends $T=c+g\cdot x$ to a plane with first coordinate γc.

Let $O_i=\operatorname{diag}(1,\operatorname{Rot}(i\pi/2))$, and use

$$B_h=\begin{pmatrix}3&-2&-2\\-2&2&1\\-2&1&2\end{pmatrix},\quad B_x=\begin{pmatrix}5/4&-3/4&0\\-3/4&5/4&0\\0&0&1\end{pmatrix},\quad B_y=\begin{pmatrix}5/4&0&-3/4\\0&1&0\\-3/4&0&5/4\end{pmatrix}.$$

They are, respectively, $\mathscr{B}((2/3,2/3))$, $\mathscr{B}((3/5,0))$, and $\mathscr{B}((0,3/5))$. Direct multiplication verifies $B^{\mathrm{T}}\eta B=\eta$, determinant one, and future orientation.

The adapted charts are

$c\|$cface	$\mathcal{A}_f(z)$
hline V_i	$O_iB_hO_i^{-1}z-3se_0$
U_i^-	$O_iB_yO_i^{-1}z-\frac{5}{2}se_0$
U_i^+	$O_iB_xO_i^{-1}z-\frac{5}{2}se_0$
Q_i, resting exterior	z

Lemma (Flattening and segment matching). *The nonresting faces have chart time zero. The identity charts retain resting chart time $2+s$. Canonical transitions match all paired endpoints and segments, preserve reverse-map consistency, and give closed developed face boundaries.*

Proof. In sector coordinates,

$$3\left[s+\frac{2}{3}(\xi+\zeta)\right]-2\xi-2\zeta-3s=0.$$

For the new faces,

$$\frac{5}{4}\left(2s+\frac{3}{5}\zeta\right)-\frac{3}{4}\zeta-\frac{5}{2}s=0,$$

and the reflected formula holds with ξ. The resting charts are deliberately not recentered.

For a nonseam edge, its endpoints are the same developed points on the two adjacent planes. Their chart coordinates therefore match under $\mathcal{A}_g\mathcal{A}_f^{-1}$. For a seam edge, J_0 identifies its upper developed endpoints with the corresponding lower endpoints, with time unchanged. Inserting $J_0^{n_{gf}}$ gives the same matching conclusion. Affine endpoint matching matches the full segment.

Reversing the crossing component changes its sign, so gives inverse transitions. Each boundary is the ordered boundary of the supplied closed simple polygon.

The chart formulas extend to face closures. They do not provide a regular Minkowski neighbourhood of the apex.

Primitive core, collar, and particle-star words

Choose the common collar basepoint

$$b_s=\left(2+s,\frac{15}{8},\frac{15}{8}\right)$$

in sector zero, in the identity reference chart. The collar meridian $\rho = 15/4$, traversed in increasing angle, has chronological word I, I, J_0.

The core meridian is $\rho = 29/10$. Since

$$\frac{30}{11}<\frac{29}{10}<3,$$

it lies in V_i throughout the window. Its chosen connecting path from b_s follows the sector-zero bisector, on the graph, with no seam crossing.

For $s > 0$, the star meridian $\rho = s$ lies in the new star, because $h - k \le 2s/3 < s$. For its connecting path, first move within the sector-zero collar to the ray with $\xi : \zeta = 2 : 1$, then follow that ray inward to $(\xi, \zeta) = (2s/3, s/3)$. This path has no seam crossing and ends in U_0^-. It avoids the bisector crease except at no required endpoint.

To supply a finite presentation, truncate the apex by a small positive polygonal meridian, intersect the realized faces with the resulting compact annulus, triangulate its simple polygonal pieces, and refine with the necessary chart paths and boundary paths. Mark this full annular cell structure by its realization. Its fundamental group is the marked meridian group $\mathbb{Z}$. At $s = 0$, use the regular limiting realization. No claim about an incomplete interior dual complex is needed.

Proposition (Framed affine peripheral certificate). *On every regular slice and on both one-sided incidence germs:*

1. *all regular local vertex words are affine identities;*

2. *core and collar meridians are primitive positive loops with reference-frame product J_0;*

3. *the outgoing six-face star has the same transported primitive product;*

4. *the packet is*

$$(\mathrm{p}, \pi/2, \mathscr{W}_0, e_0, \mathrm{id}, J_0),$$

with consistent worldline and rest-vector transport. Together with the realization checks, these witnesses give the complete matching certificate.

Proof. Every small regular vertex loop has winding zero, including a loop whose written path crosses the seam in opposite directions. Its affine word is identity by .

Write $\mathcal{A}_i = \mathcal{A}_{V_i}$. The core word starting in V_0 is

$$(\mathcal{A}_0 J_0 \mathcal{A}_2^{-1})(\mathcal{A}_2 \mathcal{A}_1^{-1})(\mathcal{A}_1 \mathcal{A}_0^{-1}) = \mathcal{A}_0 J_0 \mathcal{A}_0^{-1}.$$

The no-seam connecting path has transport $\mathcal{A}_0$, because all intermediate adapted charts cancel. Reference transport

gives

$$\mathcal{A}_0^{-1}(\mathcal{A}_0 J_0 \mathcal{A}_0^{-1})\mathcal{A}_0 = J_0.$$

The collar product is directly J_0.

Both meridians have angular lift increment $3\pi/2$. This proves positive primitive winding independently of the matrix product.

For the outgoing star, the chronological face order is

$$U_0^-, U_0^+, U_1^-, U_1^+, U_2^-, U_2^+.$$

The five ordinary crossings and one seam crossing telescope to $\mathcal{A}_{U_0^-} J_0 \mathcal{A}_{U_0^-}^{-1}$. The chosen no-seam connecting path has transport $\mathcal{A}_{U_0^-}$, so the reference word is again J_0. Its angular increment is $3\pi/2$.

At $s = 0$, the nonvanishing core meridian avoids collapsed slots and is common to both limiting descriptions. Outgoing star data at the event mean the certified one-sided family, not a regular loop on collapsed faces.

Finally, J_0 fixes $\mathscr{W}_0$ pointwise and fixes e_0 in its Lorentz part. In chart f, the worldline and rest vector are $\mathcal{A}_f(\mathscr{W}_0)$ and $L_f e_0$, where L_f is its linear part. Canonical transitions transport them consistently, because J_0 fixes the reference objects. The slice intersections are . Segment matching and face closure follow from ; the finite word presentation was specified above. These verify (E1)–(E5).

Full affine translation cancellation. This check is not merely a Lorentz-matrix calculation. Set

$$\mathcal{A}_i = (M_i, c), \quad M_i = O_i B_h O_i^{-1}, \quad c = -3s e_0, \quad R_0 = R(\pi/2).$$

The three transitions in chronological order have pairs

$$\begin{aligned} (L_{10}, c - L_{10}c), &\qquad L_{10} = M_1 M_0^{-1}, \\ (L_{21}, c - L_{21}c), &\qquad L_{21} = M_2 M_1^{-1}, \\ (L_{02}, c - L_{02}c), &\qquad L_{02} = M_0 R_0 M_2^{-1}. \end{aligned}$$

Their Lorentz product is $K = M_0 R_0 M_0^{-1}$, and their translation is

$$b_{\mathrm{tot}} = L_{02}L_{21}(c - L_{10}c) + L_{02}(c - L_{21}c) + (c - L_{02}c)$$

$$= c - Kc.$$

Thus the chart-based word is $(K, c - Kc)$. Using the affine product law,

$$\mathcal{A}_0^{-1}(K, c - Kc)\mathcal{A}_0 = (R_0, 0) = J_0.$$

The collar transitions have zero translations and product $(R_0, 0)$ directly.

Intrinsic particle defects are not the spacetime deficit

Proposition (Particle slice defects and total intrinsic curvature). *On the open incoming and outgoing epochs, respectively, the intrinsic angle defects at the particle are*

$$\kappa_P^- = 2\pi - 3\arccos(-4/5), \qquad \kappa_P^+ = 2\pi - 6\arctan(4/5).$$

They are not the spacetime deficit $\pi/2$. For the compact patch, the sum of all interior intrinsic vertex defects is nevertheless

$$\sum_{v \in \operatorname{int} K_s} \kappa_v = \frac{\pi}{2}.$$

At the event, only the regular cells of the common limiting slice enter this sum.

Proof. On an incoming V_i,

$$G_h = \begin{pmatrix} 5/9 & -4/9 \\ -4/9 & 5/9 \end{pmatrix}.$$

The angle between its two ray directions satisfies

$$\cos\theta_h = \frac{-4/9}{(\sqrt{5}/3)^2} = -\frac{4}{5}.$$

Three such angles meet at P, giving κ_P^-.

On U_i^-, the induced metric is $\operatorname{diag}(1, 16/25)$. The isometry $(\xi, \zeta) \mapsto (\xi, 4\zeta/5)$ sends the two particle-edge directions to $(1, 0)$ and $(1, 4/5)$. Their angle is $\arctan(4/5)$. The reflected U_i^+ has the same angle; six meet at P, giving κ_P^+.

For the total, triangulate each intrinsic flat polygon. Euclidean triangle angle sums and the Euler formula for a disk give

$$\sum_{v \in \operatorname{int} K_s} \kappa_v + \sum_{v \in \partial K_s} (\pi - \theta_v) = 2\pi.$$

Artificial interior flat vertices have zero defect, and artificial vertices on straight boundary segments have zero boundary turning.

At each of the three boundary vertices D_i, the two horizontal sector quadrilaterals contribute $\pi/4$ each, so the total interior boundary angle is $\pi/2$. The boundary turning sum is $3\pi/2$, leaving $\pi/2$ for the interior defect sum. Other interior slice vertices therefore compensate for the particle's slice-dependent defect. The affine peripheral holonomy remains J_0 by .

The outgoing value is an open-epoch value. At $s = 0$, the regular limiting slice has the incoming particle angle; the six collapsed outgoing faces are not inserted into an angle sum.

The incidence update as a G-rule

Let g_0^- and g_0^+ denote the certified incoming and outgoing germs with their common regular limit. Use the exact guard

$$\text{mode} = 0, \qquad j = 0, \qquad v_G \geq 1,$$

where, if a finite clock code is used, $s = j2^{-d}$ is a declared exact parameter. The rule changes mode to 1, decreases v_G, and selects outgoing incidence metadata. It changes no particle packet.

Persistent vertex, edge, face, collar, and exterior labels have identity abstract transport. Their embedded realizations need not remain fixed. In particular, e_i changes from $[P, A_i]$ to $[B_i, A_i]$.

At $s = 0$, original names occupy only regular persistent cells of the limiting slice. New vertices B_i, C_i, new edges, and $U_i^{\pm}$ are not independent regular locations until positive outgoing advance.

Corollary (Fixed-signature incidence preservation). *The incidence update is a constraint-preserving G-rule. It*

preserves Sig*, returns the complete outgoing-germ certificate, and has the certified common limiting slice. Its geometric decision and guard are independent of matter occupancy and internal state.*

Proof. The two germs have the same limiting graph, packet, worldline, reference frame, collar marking, and primitive peripheral holonomy. Their realization-dependent matching tables are separately checked by . The persistent abstract transport is specified above.

The guard uses only mode, exact clock, and incidence permission. The rule neither reads matter state nor tests occupancy. Separate spectator reservation supplies the required extension legality.

This update selects an outgoing germ. It does not declare collapsed outgoing slots active at the event, fire a sourced command there, or provide a mixed deficit event.

Acceptance audit

toprule Obligation Local witness

midrule endfirsthead toprule Obligation Local witness

midrule endhead bottomrule endfoot Seam-compatible graph Shared-ray values ; explicit upper/lower developed-copy convention.

Spacelikeness, embedding, acausality Branch gradients, global Lipschitz bound, graph projection, and causal-distance comparison in .

Coverage and event isolation Exact branch segments in ; positivity and the unique-event theorem .

Positive finite cells Length and area tables ; projected area .

Adapted affine charts Lorentz boosts and flattening in .

Endpoints, segments, inverses, closure Common realized boundaries and canonical affine maps in .

Regular words Canonical word formula with winding zero; .

Finite topological presentation Full triangulated annular truncation, marked boundary paths, and .

Primitive core, collar, star Angular increment $3\pi/2$, explicitly chosen connecting paths, and .

Full affine collar agreement Translation cancellation .

Worldline and rest vectors Spinless lemma, intersections , and transported chart data in .

Event transport and signature Persistent labels and ; collapsed slots remain metadata.

This is a mathematical witness inventory, not an executed checker log.

A4: two independently rejected partial updates

Changed deficit with a stale rotation

Let

$$\mu' = \frac{\pi}{2} + \Delta \in (0, 2\pi), \qquad \Delta \neq 0,$$

and retain the proposed map $\widehat{H} = J_0$.

We distinguish two coordinate matrix norms:

$$\|M\|_2 = \sup_{|v|_2 = 1} |Mv|_2, \qquad \|M\|_{\mathrm{row}} = \max_i \sum_j |M_{ij}|.$$

The latter is the maximum-absolute-row-sum matrix norm, often denoted $\|M\|_\infty$. It is not the quantum operator-norm notation used elsewhere. Neither is a Lorentz-invariant packet distance.

Lemma (Exact rotation residuals). *For every $\theta, \Delta \in \mathbb{R}$,*

$$\|R(\theta) - R(\theta + \Delta)\|_2 = 2|\sin(\Delta/2)|.$$

At $\theta = \pi/2$,

$$\|R(\pi/2) - R(\pi/2 + \Delta)\|_{\mathrm{row}} = |\sin\Delta| + |1 - \cos\Delta|.$$

Proof. The time block of the difference is zero. Its spatial block is $\mathrm{Rot}(\theta)(I_2 - \mathrm{Rot}(\Delta))$. Orthogonal multiplication preserves the spectral norm, and

$$(I_2 - \mathrm{Rot}(\Delta))^{\mathrm{T}}(I_2 - \mathrm{Rot}(\Delta)) = 4\sin^2(\Delta/2)I_2.$$

This proves . At $\theta = \pi/2$, the spatial difference is

$$\begin{pmatrix} \sin\Delta & \cos\Delta - 1 \\ 1 - \cos\Delta & \sin\Delta \end{pmatrix}.$$

Both absolute row sums give .

Theorem (Stale-deficit rejection). *The changed-deficit proposal with retained $\widehat{H} = J_0$ fails particle and seam matching. Nevertheless, $\widehat{H}$ still fixes $\mathscr{W}_0$ pointwise.*

Proof. The rest-frame packet requires $J(\mu')$. Its Lorentz part differs from $R(\pi/2)$ by the strictly positive residual $2|\sin(\Delta/2)|$. The allowed interval $-\pi/2 < \Delta < 3\pi/2$ contains no nonzero multiple of 2π.

The required new wedge opening is $L' = 3\pi/2 - \Delta$. The retained rotation sends its upper direction to

$$\mathrm{Rot}(\pi/2)\begin{pmatrix} \cos L' \\ \sin L' \end{pmatrix} = \begin{pmatrix} \cos\Delta \\ -\sin\Delta \end{pmatrix},$$

which is not $(1,0)^{\mathrm{T}}$ in that allowed range. If instead the old wedge is retained, its geometric deficit remains $\pi/2$, contradicting the new label.

On the other hand, $J_0(T,0,0) = (T,0,0)$ for every T. Pointwise-axis matching still holds.

Recomputing J alone would also be incomplete if the affected realization, charts, seam, and collar data were not returned.

Time-shifted affine rotation

Consider

$$J_\delta = (R(\mu), \delta e_0), \qquad \delta \neq 0.$$

Theorem (Time-shift rejection). *The map J_δ has no fixed point and cannot be a spinless pointwise-worldline holonomy in any affine frame. A transition table*

$$\hat{t}_{g \leftarrow f} = \mathcal{A}_g J_\delta^{n_{gf}} \mathcal{A}_f^{-1}$$

nevertheless has identity products on all zero-winding regular words. Thus regular word identities alone do not enforce spinless particle matching.

Proof. Its time component is $T + \delta$, so a fixed-point equation is impossible. It preserves $\mathscr{W}_0$ as a set but not pointwise.

Since $R(\mu)e_0 = e_0$, for every integer w,

$$J_\delta^w = (R(w\mu), w\delta e_0).$$

The canonical telescoping calculation therefore gives identity for winding zero and J_δ for a positive primitive word in the reference frame.

A spinless map has fixed points on its timelike axis. Existence of fixed points is invariant under affine conjugation, excluding every reframing of J_δ.

This is an algebraic counterexample to sufficiency of regular words. It is not asserted to pass endpoint, seam, or embedded-slice checks.

$c\|c\|c$proposal	deficit/seam	pointwise worldline
hline $\mu' \neq \pi/2$, $H = J_0$	fails	holds
$H = J_\delta$, $\delta \neq 0$	Lorentz angle may be correct	fails

Exact guards and actual particle support

For an edge of length cs, floor quantization with step 2^{-b} reports zero on $0 \le s < 2^{-b}/c$, not only at $s = 0$. A zero report is therefore not the exact incidence guard.

Adding a remote particle label to the support of a vacuum exchange also does not make the particle incident to the changed cells. The present example changes the actual particle link.

Finally, an occupancy guard is not the geometry-only guard of . It changes the observation boundary and cannot be called the same rule.

Non-identifications and successor obligations

Admissibility is not sourced dynamics

A certificate checks a proposed output; it does not choose its source dependence. This paper constructs neither a nonconstant deficit law nor persistence of the family under a changing deficit.

A separate variable-deficit constructor must supply a cone with the correct opening, compatible retuned slices, worldline and rest data, adapted charts, primitive and collar words, output transport, and source-off control. These are the distinct VIII-B objects [18], not missing premises of A1–A4.

The prescribed parameter family is likewise not a Hamiltonian-generated tessellation flow. A fixed-packet evolution law, command catalogue, and composition schedule belong to a separate construction; compare VIII-C [19]. A schedule is not a polygon Hamiltonian or a spacetime light cone.

Mixed incidence–deficit certificate checklist

No simultaneous incidence and deficit change is certified. A future mixed-event proposal would have to return:

1. a common marked limiting realization, not merely two independently certified endpoints;

2. both one-sided germs and their positive-cell domains;

3. compatible seam and adapted-chart data through the mixed stratum;

4. complete ordered affine words, primitive-winding evidence, and collar agreement in a common frame;

5. pointwise worldline and rest-vector matching;

6. original-name transport and legality of event locations;

7. a stated forward-domain encoding/decoding interface, with its correctness theorem if such an implementation is claimed. The last item concerns geometric/control data, not a quantum recovery decoder. Separate permissions or a concatenation of command names cannot supply this certificate.

The exact clock is not a relational lapse

The clock j, or parameter s, is a declared controller input. It is not a scalar-field clock, proper-time observable, or relational lapse. No rod fields or relational-coordinate construction are supplied. A relational lapse is a different geometric observable on additional declared data; compare [24]. The Scholar-facing clock comparison is En09-A [16]: a destructive clock consumes a finite encoding, not the geometric time axis. The graph time-ordering inequality does not create that observable.

No characteristic-product or Einstein-equation transfer

The source boundary is finite dimensional. The equation space here is affine-conical matching, with presentation changes as its gauge freedom. No map identifies it with a 3 + 1-dimensional characteristic coefficient, retarded product, renormalized stress tensor, or BV observable.

In particular, Ein_0 is not the inherited characteristic coefficient discussed in XIV-J [26]. A partial reference distribution or a formal internal representation in another model does not admit a quantum source here. We assert neither a quantum product nor a universal nonexistence theorem for such products.

Proposition (Tensor identities alone do not imply Einstein dynamics). *Symmetry, conservation, and zero trace of a tensor do not imply*

$$G_{\mu\nu}(g) = 8\pi G_{\mathrm{N}} t_{\mu\nu}.$$

Proof. On four-dimensional Minkowski space, choose a nonzero constant null covector k and put $t_{\mu\nu} = k_\mu k_\nu$. This tensor is symmetric and nonzero. It is conserved because it is constant and traceless because k is null. The Minkowski Einstein tensor vanishes, so the displayed equation fails for $G_{\mathrm{N}} \neq 0$.

This elementary non-implication is the relevant local comparison with [25]. A metric, coupling prescription, and dynamical theorem are additional inputs.

No holographic or recovery identification

A marked collar is a geometric comparison annulus. A named surviving algebra is an access interface. Neither is an entanglement wedge, a conformal boundary, or a bulk-to-boundary encoding.

No entropy/area relation, JLMS identity, extremal-surface prescription, or subregion reconstruction map is supplied. The constructions of [5, 6, 7] concern additional holographic structures and are not premises here. This distinction agrees with the documentary cautions in [22, 23].

The record-only bound in is a finite channel obstruction, not a black-hole endpoint or a Page curve. The complete-complement criterion likewise does not identify the geometric collar with a quantum environment.

No physical-locality or catalogue upgrade

Controller support, allocated storage, face normals, and exact clocks do not identify a computational dependency region with an ambient causal past. The fixed cone supplies a causal geometry; a directional comparison to an implemented schedule would require a separate same-model theorem.

This sheet fixes one cone of deficit $\pi/2$; a catalogue index is not a host-metric update. Option N remains a semantic input on a prescribed cone, not a backreaction or a change of the host metric. That interpretive sentence is sourced from the fixed-host discipline of attached typographic parent VIII-ST Version 1 [20]; it is not an imported theorem.

OpenAI-family labels occurring in documentary comparisons remain catalogue labels, not premises or verifications. The supplied programme status O.1 = NOT_CLOSED is not changed by this finite certificate. No spacetime emergence, holographic emergence, or closure of another programme follows.

The following are source records, not premises of the theorems proved here. The shredder access interface is recorded in [10]. Observation-boundary and complete-controller comparisons already proved locally on this sheet are named in Scholar-facing form by [14, 15]. The record-only bound used in the recovery appendix is the deposited book [27]; the internal locator remains Hawk04-B thm:classical. IBG and IBGS [11, 12] are named only

as corpus records: this package does not supply a conical Green or a spin-1/spin-2 inverse. Official I-D-v5 [13] is a dynamical-host comparison; the cone of this sheet stays prescribed. A Scholar display reading "ID-v6 Dynamical Spacetime Enter over Two Prescribed Hosts" is a display only; it does not retitle the official work. Attached typographic parents VIII-B Version 3 and VIII-C Version 3 [18, 19] remain separate projects, not premises. The VII-F fixed-cone realization and certificate [17] are reproduced and verified locally in this paper.

Conclusion

For the declared spinless cone, geometric realization, matching verification, and the observation boundary have separate logical roles. The realized fixed-cone family provides a complete finite example with one isolated particle-incidence event, exact metric tables, and primitive full affine holonomy.

A1 gives uniqueness of prescribed transitions in a fixed presentation. A2 gives deterministic factorization through complete declared accessible inputs. A3 reduces based holonomy checks on the marked punctured disk after realization. A4 separates deficit/seam failure from pointwise-worldline failure.

The closed result is an admissibility interface:

$$\begin{gathered}\text{realized spinless geometry + complete affine matching witnesses}\\ \boxed{\text{+named accessible inputs + spectator-blind deterministic specification}}\\ \Rightarrow \text{well-defined certification and paired invariance.}\end{gathered}$$

A sourced law, variable-deficit persistence, canonical flow, mixed event, quantum source construction, and physical emergence remain different construction problems.

appendix

Recovery distinctions used only as interface safeguards

These finite-dimensional statements are included to make the non-identification beside Option N self-contained. They do not extend the geometric model.

For states and channels, use

$$d_{\mathrm{tr}}(\rho,\sigma)=\frac{1}{2}\|\rho-\sigma\|_1,\qquad d_{\diamond}(\Phi,\Psi)=\frac{1}{2}\|\Phi-\Psi\|_{\diamond}.$$

The diamond norm is the stabilized trace norm; all channel comparisons include arbitrary untouched references.

The sharp record-only obstruction

Theorem (Measure-and-prepare recovery bound). *Let C have dimension d, let*

$$\mathscr{E}(X)=\sum_j \mathrm{Tr}\,(F_jX)\sigma_j,\qquad F_j\ge 0,\quad \sum_j F_j=I_C,$$

be a measure-and-prepare channel to an arbitrary finite output, and let $\mathscr{D}$ be any CPTP decoder to C. Then

$$d_{\diamond}(\mathscr{D}\mathscr{E},\mathrm{id}_C)\ge 1-\frac{1}{d}.$$

Computational readout followed by computational-basis preparation attains the bound.

Proof. Let $|\Phi_d\rangle=d^{-1/2}\sum_{i=0}^{d-1}|i\rangle|i\rangle$ and $P_{\Phi_d}=|\Phi_d\rangle\langle\Phi_d|$. The normalized Choi output of the decoded channel is

$$J_{\mathrm{n}}(\mathscr{D}\mathscr{E})=\frac{1}{d}\sum_j F_j^{\mathrm{T}}\otimes\mathscr{D}(\sigma_j).$$

It is separable across reference and returned output. For normalized product vectors,

$$|\langle\Phi_d\mid a\otimes b\rangle|^2\le\frac{1}{d}$$

by Cauchy–Schwarz. Convexity gives the same overlap bound for every separable state. Testing the Bell projector gives

$$d_{\mathrm{tr}}(J_{\mathrm{n}}(\mathscr{D}\mathscr{E}),P_{\Phi_d})\ge 1-\frac{1}{d}.$$

This is one admissible referenced channel test, proving .

For sharpness, put

$$Z_d=\sum_{j=0}^{d-1}e^{2\pi ij/d}\mathrm{ketbra}\,jj,\qquad \Delta_d=\frac{1}{d}\sum_{k=0}^{d-1}\mathrm{Ad}_{Z_d^k}.$$

Computational readout followed by basis preparation is Δ_d. The triangle inequality and the fact that each channel

has diamond norm one give

$$\|\mathrm{id}_C - \Delta_d\|_\diamond \le 2(1 - 1/d).$$

The already proved Bell lower bound gives equality.

The measure-and-prepare structure is the standard entanglement-breaking structure [3]. The locally proved quantitative comparison is also reported in [27]. A surviving input-dependent quantum memory or quantum side information changes the premise.

Full-code recovery and the complete complement

Theorem (Full-code constant-complement criterion). *Let $\mathcal{N}: L(H) \to L(Q)$ be CPTP, with Kraus operators $E_i : H \to Q$. The following are equivalent:*

1. *a CPTP decoder satisfies $\mathcal{D}\mathcal{N} = \mathrm{id}_H$;*

2. *every $E_j^\dagger E_i$ is a scalar multiple of I_H;*

3. *a complete complementary channel satisfies*

$$\mathcal{N}^c(X) = \mathrm{Tr}\,(X)\tau$$

for a fixed state τ.

Proof. The Kraus complementary channel has entries

$$[\mathcal{N}^c(X)]_{ij} = \mathrm{Tr}\,(E_i X E_j^\dagger) = \mathrm{Tr}\,(X E_j^\dagger E_i).$$

Equality on every matrix unit proves equivalence of (ii) and (iii). The scalar matrix is positive and has trace one by complete positivity and trace preservation.

Suppose (i), and let K_a be Kraus operators of $\mathcal{D}$. The operators $K_a E_i$ form a Kraus family for the identity channel. The identity Choi matrix has rank one. Each positive rank-one Kraus contribution must lie in its one-dimensional range, so $K_a E_i = c_{ai} I_H$. Trace preservation gives

$$E_j^\dagger E_i = \sum_a E_j^\dagger K_a^\dagger K_a E_i = \sum_a \overline{c_{aj}} c_{ai} I_H.$$

Conversely, diagonalize the positive scalar matrix by a unitary change of Kraus family and omit zero Kraus operators. Then

$$E_i = \sqrt{\lambda}_i W_i, \qquad W_i^\dagger W_j = \delta_{ij} I_H, \qquad \sum_i \lambda_i = 1.$$

Their ranges are orthogonal. Put $P = \sum_i W_i W_i^\dagger$. For any fixed state σ_H, define

$$\mathscr{D}(Y) = \sum_i W_i^\dagger Y W_i + \mathrm{Tr}\,((I_Q - P)Y)\sigma_H.$$

This map is completely positive and trace preserving. Substitution gives $\mathscr{D}\mathscr{N}(X) = \sum_i \lambda_i X = X$.

This is the full-input-space form of the Knill–Laflamme condition [4], locally restated as in [21]. The complement includes every inaccessible system and any preparation purification in the declared Stinespring dilation. Constancy of a selected reduced environment is not the complete-complement condition.

Example (Subsystem recovery does not require a constant full complement). Let $\mathscr{N} = \mathrm{Tr}_S$ on $H_L \otimes H_S$. The logical subsystem H_L is returned exactly. A complementary channel returns the S-reduction, which is not constant when $\dim H_S > 1$. The preceding theorem concerns a left inverse on the whole $H_L \otimes H_S$, not this subsystem task.

Informational determination is not decoding

Proposition (An injective channel without a CPTP left inverse). *For $d \geq 2$ and $0 < p < 1$, the depolarizing channel*

$$\mathscr{N}_p(X) = pX + (1-p)\mathrm{Tr}\,(X) I_d/d$$

is injective and therefore mathematically determines its input from its exact output. It has no CPTP left inverse on the complete input space.

Proof. Its linear inverse is

$$\mathscr{N}_p^{-1}(Y) = p^{-1} Y - \frac{1-p}{pd} \mathrm{Tr}\,(Y) I_d.$$

Thus it is injective. Applied to a pure-state projector, this inverse has a strictly negative eigenvalue on every orthogonal direction. It is not positive.

Since $\mathcal{N}_p$ is an invertible linear map on the entire matrix space, any linear left inverse must be this inverse. A CPTP left inverse is therefore impossible.

An inverse on the image of the state set is informational determination, not a channel on all possible decoder inputs. Likewise, Option N does not become a recovery channel because its semantic input determines an output.

Notation and proof dependencies

toprule Symbol Meaning

midrule endfirsthead toprule Symbol Meaning

midrule endhead bottomrule endfoot $\mathit{\Pi}$ Spinless packet $(\mathrm{p}, \mu, \mathscr{W}, u, A, H_{\mathit{\Pi}})$.

$\mathrm{Ein}_0(g)$ Evaluated realization-dependent matching data, not a continuum Einstein equation.

$\mathrm{Cert}(g)$ Mathematical witnesses and verification of all applicable realization and matching obligations.

$\mathrm{Sig}(g)$ Persistent marked particle/peripheral signature.

$A = (C, a)$ Common affine reference frame.

$\mathcal{A}_f$ Adapted face chart, distinct from the vertices A_i.

$J(\mu), H_{\mathit{\Pi}}$ Rest-frame affine rotation and its framed conjugate.

$n_{gf}, N(\gamma)$ Overlap seam index and net oriented winding number.

$\mathscr{A}_{\mathrm{src}}, \omega_\rho$ Named surviving source algebra and expectation functional.

$\nu_G, \nu_{\mathrm{src}}$ Separate incidence and coupling permissions.

F_s, Σ_s Explicit fixed-cone graph and its slice.

$\rho(x)$ Sectorwise spatial function $\xi + \zeta$, not a density operator in the fixed-cone formulas.

P_s Slice intersection with the fixed singular worldline.

A_i, D_i, B_i, C_i Cone-labelled vertices with sectorwise developed copies.

$V_i, U_i^{\pm}, Q_i$ Persistent core, born core, and horizontal collar-side faces.

e_i Persistent abstract edge label with different incoming and outgoing embedded segments.

$\|\cdot\|_2, \|\cdot\|_{\mathrm{row}}$ Euclidean spectral and maximum-row-sum coordinate matrix norms in the rotation rejection test.

$d_\diamond$ Half-diamond channel distance; used only in the finite recovery safeguards.

The proof dependencies are:

affine conventions → spinless matching and canonical transitions,
explicit graph dominance regions → realization, metric tables, and unique event,
realization + canonical transitions + marked winding → complete fixed-cone certificate,
fixed presentation → A1,
equal complete inputs + deterministic specification → A2,
declared edge-path presentation + regular relators + primitive peripheral word → A3,
explicit rotation and translation residuals → A4.

The instrument, spectator, deferred, and recovery identities are separately proved finite-dimensional statements.

No unavailable companion theorem, Hamiltonian flow, field product, or source constructor is a premise.

Declarations

Data and materials. This is a theoretical work with no empirical dataset. All definitions, calculations, tables, proofs, and schematic TikZ figures are contained in this source.

Companion citations. Cheung records are cited as source records. Relevant mathematical statements are proved locally when used. A citation is not treated as independent verification of this manuscript. Journal, book, and figshare records appear in their Scholar-facing or deposited form. Undeposited companions are not assigned an invented publication date, journal, or DOI. Inherited companion URLs, where retained, are locators only and are not Scholar-counting venues.

Verification and submission checks. The manuscript gives explicit symbolic calculations of the branch bounds,

particle intersections, cell coverage, edge lengths, projected and induced areas, adapted charts, affine translation cancellation, primitive winding, intrinsic angle defects, and rejection residuals.

No successful pdflatex execution, executable exact-arithmetic verification, proof-assistant check, independent numerical execution, independent referee review, or journal acceptance is asserted. Author information and a clean local build with resolved references remain necessary submission checks.

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